Revealing the mathematical potential of special needs students

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Universiteit Utrecht

Freudenthal Institute, Faculty of Science
Compulsory education

Primary education
K1-2 and Grades 1-6

VWO

HAVO

Higher Vocational Education

MBO (Intermediate Vocational Education)

VMBO

Basic Vocational Strand

Special Secondary Education

University

Special Primary Education

3%
Overview

Reform in mathematics education

Suitable for SE students?

Focus is too much on what SE students *do not* know instead on what they *do* know

Assessing SE students mathematical potential

**Study 1**
Can SE students solve ratio problems?

**Study 2**
Can SE students use various solution strategies?

**Study 3**
Can SE students solve combinatorics problems?

**Study 4**
Can we open SE students’ ZPD by offering them optional auxiliary tools?

FaSMEd Project
A digital assessment environment for formative assessment
Realistic Mathematics Education

“realistic”

• ZICH REALISEREN = to imagine

• meaningful context
  • real world or fantasy world
  • formal world of mathematics

~1968

2016

• still under construction
• over the years different accentuations
# Realistic Mathematics Education

**Mechanistic Mathematics Education**

- teaching is transmission
  * atomized
  * step-by-step
- bare number calculations
- little attention applications
  (especially not at the start)
- fixed procedures, recipes
- directly at the formal level

- distinct strands
- mostly individual work
- much guidance

## Activity Principle

- activity principle

## Reality Principle

- reality principle

## Level Principle

- level principle
  * various levels of understanding
  * progressive schematization
  * models as bridges

## Intertwinement Principle

- intertwininement principle

## Interactivity Principle

- interactivity principle

## Guidance Principle

- guidance principle
Often heard opinion:
A reformed approach to mathematics education (such as *Realistic Mathematics Education*) is not suitable for Special Education students who are weak in mathematics.

- Starting from contexts
- Building on children’s informal knowledge
- Various solution strategies
- Reflection on solution strategies
Proof that SE students are poor in mathematics

Offering SE students a limited mathematics program

Avoiding reformed teaching methods

Low scores on standardized mathematics tests

“Proof” that SE students are poor in mathematics
Often heard opinion:
A reformed approach to mathematics education (such as *Realistic Mathematics Education*) is **not suitable for Special Education students** who are weak in mathematics

- Starting from contexts
- Building on children’s informal knowledge
- Various solution strategies
- Reflection on solution strategies

It is time to reveal what SE students *know*, rather than what they do *not* know

This implies a change in assessment:
Assessing SE students’ mathematics potential
6 A test on ratio – what a paper-and-pencil test can tell about the mathematical abilities of special education students

6.1 Introduction

In The Netherlands, in addition to regular schools for primary education, there are also schools for special education. The system of special education comprises fourteen kinds of schools, which are attended by some 5% of primary school-age children. Among these are schools for children with learning and behavioral difficulties, for mildly mentally retarded children, severely mentally retarded children, deaf children, the blind and visually handicapped, and for children with severe emotional problems.

Two kinds of schools, that is, for children with learning and behavioral difficulties and for mildly mentally retarded children, account for the great majority of these children. Some three-quarters of the children in special education attend one of these two types of schools.
Research question
- Can students in special education solve ratio problems?

Method
- Participants
  - 61 students
  - in 4 upper-grade classes
  - from 2 schools for mildly mentally retarded students
  - 10.5 to 13 years old
  - mathematics level at Grades 2 to 4 of regular school
    (2 to 4 years behind)
### Study 1

<table>
<thead>
<tr>
<th></th>
<th>School 1</th>
<th></th>
<th>School 2</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Class 1</td>
<td>Class 2</td>
<td>Class 1</td>
<td>Class 2</td>
</tr>
<tr>
<td>(mental) arithmetic to 20</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>(mental) arithmetic to 100</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
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<td>column addition/subtraction</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>column multiplication</td>
<td>x</td>
<td>x</td>
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<tr>
<td>column division</td>
<td>x</td>
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<tr>
<td>decimal numbers</td>
<td></td>
<td></td>
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<tr>
<td>ratio</td>
<td></td>
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</tr>
<tr>
<td>geometry</td>
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<td></td>
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<td>metric system</td>
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<td>arithmetic with money</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>other</td>
<td></td>
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</tbody>
</table>
Research question
- Can students in special education solve ratio problems?

Method
- Participants
  - 61 students
  - in 4 upper-grade classes
  - from 2 schools for mildly mentally retarded students
  - 10.5 to 13 years old
  - mathematics level at Grades 2 to 4 of regular school (2 to 4 years behind)
- Test on ratio
  - 16 problems
  - 4 types of ratio problems
    - finding the ratio
    - comparing ratios
    - producing equivalent ratios
    - finding the fourth proportional
  - class-administered with oral instruction
<table>
<thead>
<tr>
<th>Study 1</th>
<th>Non-numerical</th>
<th>Non-numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Producing equivalent ratios</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>Yellow</td>
<td>30 minutes</td>
</tr>
<tr>
<td></td>
<td>Blue</td>
<td></td>
</tr>
<tr>
<td>Make a lot more green paint</td>
<td></td>
<td>Draw a different walk and fill in how many minutes it will take</td>
</tr>
<tr>
<td>Finding the fourth proportional</td>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>Draw the ladybird</td>
<td>How much do the three glasses of lemonade cost?</td>
</tr>
</tbody>
</table>
## Results

<table>
<thead>
<tr>
<th>Item</th>
<th>% correct</th>
<th>Item</th>
<th>% correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. pen</td>
<td>39 (+10)</td>
<td>8. paint</td>
<td>43</td>
</tr>
<tr>
<td>2. paper clip</td>
<td>28 (+23)</td>
<td>9. coin dispencer</td>
<td>44</td>
</tr>
<tr>
<td>3. road sign</td>
<td>13 (+48)</td>
<td>10. walk</td>
<td>38</td>
</tr>
<tr>
<td>4. tree</td>
<td>57</td>
<td>11. ladybird</td>
<td>64</td>
</tr>
<tr>
<td>5. lemonade</td>
<td>54</td>
<td>12. string of beads</td>
<td>51</td>
</tr>
<tr>
<td>6. toothpaste</td>
<td>30</td>
<td>13. steps</td>
<td>38</td>
</tr>
<tr>
<td>7. swimming</td>
<td>44</td>
<td>14. glasses</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>15. newspapers</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16. lpg</td>
<td>13</td>
</tr>
</tbody>
</table>
Producing equivalent ratios – student work

Study 1

Walk item: p-value .38
Finding the fourth proportional – student work

Glasses item: $p$-value \(0.64\)
Finding the fourth proportional – student work

News papers item: p-value .26
Finding the fourth proportional – student work

$Lpg$ item: $p$-value .13
Study 1

Found results & Expected results

Special Educationalist1
Special Educationalist2
Special Education Inspector1
Special Education Inspector2

Teacher Class 2.2
Teacher Class 1.2

P-value Class 2.2
P-value Class 1.2

Walk item: p-value .38
Current situation

The response of special education traditionally has been behavioral. We use task analysis, breaking up problem solving into steps. We use mnemonics and flash cards to aid with fact memorization and retrieval. We teach algorithms for solving each type of problem and then use various drill-and-practice methods. We emphasize skills instruction, starting with the most basic skills and steps and progress to the more complex (Woodward & Montague, 2002). Several prominent reviews of the literature in learning disabilities (LD) support the use of these strategies (e.g., Vaughn, Gersten, & Chard, 2000).

Cole & Wasburn-Moses (2010)
In: Teaching Exceptional Children
There are not many controlled studies

The studies that are carried out suggests that the principles of direct or explicit instruction can be useful

… including generic problem solving strategies and more classic direct instruction approaches where students are taught one way to solve a problems and are provided with extensive practice.

Baker et al. (2002). A synthesis of empirical research on teaching mathematics to low-achieving students.

A balanced approach is needed, but …

**explicit instruction** with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation (p. 425)
Hint:
Put one’s trust in self-discovery is trusting on something that failed before (Ruijssenaars, 1992). Letting students who are weak in mathematics discovering strategies by themselves is fatal. Lead them by the hand, tell them which strategies they have to use, and show them how the strategy works. Show them how to do! (p. 36)
pupils at special schools for primary education can best learn arithmetic using one specific strategy. When adding and subtracting with numbers less than 100, these pupils make least mistakes when using the so-called threading strategy (for example, 65 - 23 = 65 - 20 - 3).

Bauke Milo investigated how children with learning difficulties can best learn to add and subtract numbers less than 100. Arithmetic skills are modern methods challenge pupils to come up with their own solutions. However, children with learning difficulties require a different approach. The skills expected in modern arithmetic education are out of reach of many children at special schools for primary education.
Disadvantages of one fixed solution method

- **Prevents SE students to develop numeracy**, which is seen as important for all citizens (Warry et al. 1992); because numeracy implies that students should be able to choose a suitable method when solving number problems (Treffers, 1989; Van den Heuvel-Panhuizen, 2001)

- **Requires an unnecessary long solution path**
  (see e.g., Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009)

- **Implies ‘didactical ballast’ for students**
  (Van den Heuvel-Panhuizen, 1986)
Research question

- Can SE students use various solution strategies?
  Can SE students make spontaneous use an adding-on strategy for solving subtraction problems up to 100?

Method

- Participants
  - 56 students
  - from 14 classes
  - from 3 SE schools
  - 8–12 years old ($M=10.5; \ SD=10.4 \text{ months}$)
  - mathematics level: Cito LOVS End Grade 2 (< Grade 2 regular school)

- Computer-based test on subtraction up to 100
  - 15 items
  - with various number and format characteristics
<table>
<thead>
<tr>
<th>Number characteristics</th>
<th>Format characteristics</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Bare number</td>
</tr>
<tr>
<td>Difference between numbers</td>
<td></td>
</tr>
<tr>
<td>Crossing ten</td>
<td></td>
</tr>
<tr>
<td>Numbers close ten</td>
<td></td>
</tr>
<tr>
<td>Example</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type</th>
<th>Difference between numbers</th>
<th>Crossing ten</th>
<th>Numbers close ten</th>
<th>Example</th>
<th>Bare number</th>
<th>Taking</th>
<th>Adding</th>
<th>on</th>
<th>Total</th>
</tr>
</thead>
<tbody>
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<td>A</td>
<td>Small (&lt;7)</td>
<td>No</td>
<td>No</td>
<td>56-52</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>Small (&lt;7)</td>
<td>Yes</td>
<td>Yes</td>
<td>31-29</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>Large (&gt;11)</td>
<td>Yes</td>
<td>Yes</td>
<td>51-39</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>Large (&gt;11)</td>
<td>No</td>
<td>No</td>
<td>47-15</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>Large (&gt;11)</td>
<td>Yes</td>
<td>No</td>
<td>56-28</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

Number of items
now 29 euro discount

62 euro
space for 51 cards

49 are already included
AO use and numbers involved

Study 2

AO use and numbers involved

- 56-52
- 31-29
- 51-39
- 47-15
- 56-28
Results AO use

Data from 56 students from 14 classes showed that

- SE students can make spontaneous use of AO of 768 cases
  - DS 63%
  - AO 34%
  - Average AO use per student 4.6 (min 0, max 8)

- SE students are rather flexible in applying AO

- SE students are quite successful when applying AO
  - Adding on (260 cases): 68% correct
  - Taking away (480 cases): 51% correct
Special education students’ use of indirect addition in solving subtraction problems up to 100—A proof of the didactical potential of an ignored procedure

Marjolijn Peltenburg · Marja van den Heuvel-Panhuizen · Alexander Robitzsch

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Abstract In this study, we examined special education students’ use of indirect addition (subtraction by adding on) for solving two-digit subtraction problems. Fifty-six students (8- to 12-year-olds), with a mathematical level of end grade 2, participated in the study. They were given a computer-based test on subtraction with different types of problems. Although most students had not been taught indirect addition for solving subtraction problems, they frequently applied this procedure spontaneously. The item characteristics were the main prompt for using indirect addition. Context problems that reflect an additive frame in the task instructions were associated with a high frequency of indirect addition.
Research question

- Can special education students solve combinatorics problems?

Method

- Participants
  - 84 students (8-13 year olds; M = 11.1) from 5 SE schools
  - 76 students (7-11 year olds; M = 9.4) from 5 RE schools
  
  In each school we chose randomly for each of the CITO LOVS tests (M2, M3, M4, and M5) four students who scored near the 50th percentile score

- Instrument
  - Six combinatorics problems in ICT environment
  - Problems with the structure
    - 2 x 3, 3 x 2, 3 x 3, 2 x 2 x 2, 2 x 2 x 2, and 2 x 3 x 2
Success rate

- Overall
  - SE: 56% correctly solved
  - RE: 57% correctly solved

- Per mathematics level
Strategy use

- systematic
- semi-systematic
- non-systematic
Study 3

Frequency (%) of strategy use

SE Students

RE Students

- Non-systematic
- Semi-systematic
- Systematic

M2  M3  M4  M5

M2  M3  M4  M5
Results

Success rate

- SE students solved elementary combinatorics problems equally successful as RE students
- A significant and similar growth in success rates occurred in both school types for increasing mathematics levels

Strategy use

- SE students applied a systematic strategy equally often as RE students
- A significant increase in the use of systematic strategies occurred in both school types for increasing mathematics levels
SPECIAL EDUCATION STUDENTS’ STRATEGIES IN SOLVING ELEMNTARY COMBINATORICS PROBLEMS

Marjolijn Peltenburg, Maria van den Heuvel-Panhuizen, Alexander Robitzsch
Freudenthal Institute, the Netherlands; BIFIE, Salzburg

This paper reports on a study aimed at revealing special education students’ mathematical potential using a dynamic ICT-based assessment environment. The study focused on special education students’ (N=84) performance in the domain of elementary combinatorics; a domain which is generally not taught in primary special education. The performance of students in regular education (N=76) served as a reference. The data analysis showed that on average special education students applied a systematic strategy equally often as regular education students. Moreover, we discovered that in both school types a significant increase in the use of systematic strategies occurred.

INTRODUCTION

In general, research on supporting special education (SE) students in mathematics has a focus on the learning and teaching of basic mathematical operations, like addition and subtraction. Less attention has been paid to higher order thinking processes that go beyond standard procedural skills. This is not surprising because SE students are often
Research question

- Can we open SE students’ zone of proximal development by offering them optional auxiliary tools in digital environment?

Method

Study 1

- Participants
  - 37 students from two SE schools
  - 8-12 years old
  - mathematics level End Grade 2

- Instrument
  - 7 subtraction problems up to 100 with crossing the ten
  - In two formats
    - Standardized Cito test
    - Impulse test with optional digital auxiliary tool: 100-board
Research question

- Can we open SE students’ zone of proximal development by offering them optional auxiliary tools in digital environment?

Method

Study 2

- Participants
  - 43 students from two SE schools
  - 8-12 years old
  - mathematics level End Grade 2

- Instrument
  - 7 subtraction problems up to 100 with crossing the ten
  - In two formats
    - Standardized Cito test
    - Impulse test with optional digital auxiliary tool: empty number line
Michel’s mother has 62 euro. She buys a jacket of 58 euro. How many euro is left?
Michel’s mother has 62 euro. She buys a jacket of 58 euro. How many euro is left?
## Differences in proportion correct answers

<table>
<thead>
<tr>
<th></th>
<th>% correct answer (7 items)</th>
<th></th>
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<tbody>
<tr>
<td></td>
<td>ICT version</td>
<td>Stand. version</td>
<td></td>
</tr>
<tr>
<td>Digital manipulatives</td>
<td>54</td>
<td>34</td>
<td>(t(36)=3.67, p&lt;.01, d=.71)</td>
</tr>
<tr>
<td>(n=37)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Digital empty number</td>
<td>55</td>
<td>36</td>
<td>(t(42)=4.77, p&lt;.01, d=.75)</td>
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<tr>
<td>line (n=43)</td>
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## Students’ competence awareness

<table>
<thead>
<tr>
<th>Manipulatives study</th>
<th>Standardized version</th>
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<tbody>
<tr>
<td></td>
<td>Incorrect answer</td>
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<tr>
<td></td>
<td>Correct answer</td>
</tr>
<tr>
<td>Tool use in</td>
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<tr>
<td><em>ICT version</em></td>
<td>49%</td>
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<td>21%</td>
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<table>
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<tr>
<td></td>
<td>Correct answer</td>
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<td>Tool use in</td>
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<tr>
<td><em>ICT version</em></td>
<td>60%</td>
</tr>
<tr>
<td></td>
<td>29%</td>
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</tbody>
</table>
Results

- ICT-based dynamic assessment with optional auxiliary tools can reveal SE students’ mathematics potential

- SE students can judge their mathematical competence
ICT-based dynamic assessment to reveal special education students’ potential in mathematics

Marjolijn Peltenburga*, Marja van den Heuvel-Panhuizena and Alexander Robitzscht

aFreudenthal Institute for Science and Mathematics Education, Utrecht University, Utrecht, The Netherlands; bFederal Institute for Education Research, Innovation and Development of the Austrian School System, Salzburg, Austria

(Received 15 March 2010; final version received 31 May 2010)

This paper reports on a research project on information and communication technology (ICT)-based dynamic assessment. The project aims to reveal the mathematical potential of students in special education. The focus is on a topic that is generally recognised as rather difficult for weak students: subtraction up to 100 with crossing the ten. The students involved in the project were 8–12 years old. Their mathematical level was one to four years behind the level of their peer group in regular schools. In special education, teachers often use standardised written tests to assess their students’ mathematical understanding and computational skills. These tests do not allow students to use auxiliary resources. In the research project, an assessment instrument was developed and used in which items from a standardised written test were placed in an ICT environment. Items were enriched with an optional auxiliary tool to solve the problems. Curriculum
Formative Assessment in Science and Mathematics Education

To research the use of technology in formative assessment practices in ways that allow teachers to respond to the emerging needs of low achieving learners in mathematics and science.

NL team: Digital Assessment Environment

• Web-based
• Monitoring function
• Problems based on key competencies
• Auxiliary tools
Problem 1

When a battery is full, it will work 120 hours.
It is still charged for 40%.
For how many hours will this battery still work?
Answer: ... hours

<table>
<thead>
<tr>
<th></th>
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<td>scrap paper with grid</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>bar</td>
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<td>table</td>
</tr>
</tbody>
</table>
Problem 1

When a battery is full, it will work 120 hours. It is still charged for 40%. For how many hours will this battery still work?
Answer: \[ \ldots \] hours
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<table>
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<tbody>
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<td></td>
<td>scrap paper with grid</td>
</tr>
<tr>
<td>bar</td>
<td></td>
</tr>
</tbody>
</table>

![Bar chart showing battery charge and hours worked]
Problem 1

When a battery is full, it will work 120 hours.
It is still charged for 40%.
For how many hours will this battery still work?
Answer: 48 hours
<table>
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<th>log errors</th>
<th>log attempts count</th>
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<th>log data</th>
<th>deel-scores</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1.1 1.2 1.3</td>
<td>2.1 2.2 2.3</td>
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To conclude: We should go “across the grain”
“‘Low attaining students’ are generally classified on the basis of competence on routine tests. Perhaps it would be more accurate to say they are classified on the basis of accumulated incompetence in tests and other written work.”

(Watson, 2002, p. 461)

*Deficiency-based approach*

*Proficiency-based approach*
Watson looked for evidence that low attaining students could:

1. identify and use patterns
2. abstract through reflecting on processes
3. (counter-)exemplify and do more than imitating teacher
4. develop and use images of concepts
5. change and manipulate representations
6. perhaps work with abstractions and relations
Elvira had been asked to round 83 to the nearest 10. She replied:

“80, but if you had asked me about 87 I would have said 90.”

“In this case, the student seemed to have had an image of what it means to round numbers and had used the image to generate a counter-example in order to indicate to the teacher that she knew more than had been asked.”

3. (counter-)exemplify and do more than imitating teacher
4. develop and use images of concepts
Research on mathematical learning difficulties needs a **proficiency-based** approach.

It is time to reveal what SE students know, rather than what they do not know.

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References


References (continued)